**Truth tables**

* + - * 1. **Exercise 11**

Check the following properties for (’*nor*’), (’*nand*’) and (’*xor*’) connectives using the truth table method.

1. associativity of ’’ connective: 

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

**Theoretical result:**

**U ≡ V iff (if and only if) U and V have the same truth table.**

**(U and V are logically equivalent)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | p | q | *r* | q**↑r** | U=p **↑(q** **↑r)** | p **↑q** | V=(p **↑q)** **↑r** |  |  |  |  |  |
| i1 | T | T | T | F | T | F | T |  |  |  |  |  |
| i2 | T | T | F | T | F | F | T |  |  |  |  |  |
| i3 | T | F | T | T | F | T | F |  |  |  |  |  |
| i4 | T | F | F | T | F | T | T |  |  |  |  |  |
| i5 | F | T | T | F | T | T | F |  |  |  |  |  |
| i6 | F | T | F | T | T | T | T |  |  |  |  |  |
| i7 | F | F | T | T | T | T | F |  |  |  |  |  |
| i8 | F | F | F | T | T | T | T |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

* + - * 1. **Conclusion: U and V don’t have identical truth tables, so the associativity of nand(↑) does not hold**

**Exercise 2**

Using the truth table method decide what kind of formula (consistent, inconsistent, tautology, contingent) is . Write all the models and anti-models of .

1. 

**Theoretical results:**

1. A model of U is an interpretation of U which evaluates U as true.
2. An anti-model of U is an interpretation of U which evaluate it as false.

3. U is a consistent if it has at least one model.

4. U is valid (tautology) if all its interpretations are models.

5. U is contingent if it is consistent but not valid, if it has at least one model and one anti-model.

6. U is inconsistent if all its interpretations are anti-models.

Fp – the set of propositional formulas

S\_consistent – the set of consistent formulas

S\_contingent – the set of contingent formulas

S\_valid – the set of valid formulas (tautologies)

S\_inconsistent – the set of inconsistent formulas

Fp= S\_consistent U S\_inconsistent

S\_consistent= S\_contingent U S\_valid

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | p | q | *r* | **Ꞁp** | qV **ꞀpVr** | pVr | **Ꞁ** (pVr) | U1 |  |  |  |  |
| i1 | T | T | T | F | T | T | F | F |  |  |  |  |
| i2 | T | T | F | F | T | T | F | F |  |  |  |  |
| i3 | T | F | T | F | T | T | F | F |  |  |  |  |
| i4 | T | F | F | F | F | T | F | T |  |  |  |  |
| i5 | F | T | T | T | T | T | F | F |  |  |  |  |
| i6 | F | T | F | T | T | F | T | T |  |  |  |  |
| i7 | F | F | T | T | T | T | F | F |  |  |  |  |
| i8 | F | F | F | T | T | F | T | T |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

* + - * 1. **Conclusions:**
        2. **U1 is contingent, having 3 models and 5 anti-models:**
        3. **Models of U: i4, i6, i8:{p,q,r}->{T,F}**
        4. **i4(p)=T, i4(q)=F, i4(r)=F, i4(U)=T**
        5. **i6(p)=F, i6(q)=T, i6(r)=F, i6(U)=T**
        6. **i8(p)=F, i8(q)=F, i8(r)=F, i8(U)=T**
        7. **Anti\_models of U: i1, i2, i3, i5, i7 :{p,q,r}->{T, F}**
        8. **i1(p)=T, i1(q)=T, i1(r)=T, i1(U)=F**
        9. **i2(p)=T, i2(q)=T, i2(r)=F, i2(U)=F**
        10. **i3(p)=T, i3(q)=F, i3(r)=T, i3(U)=F**
        11. **i5(p)=F, i5(q)=T, i5(r)=T, i5(U)=F**
        12. **i7(p)=F, i7(q)=F, i7(r)=T, i7(U)=F**
        13. **Exercise 3**

Using the truth table method, check whether the following logical consequences hold:

|  |  |
| --- | --- |
|  |  |
|  |  |

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations):**

**|= (logical consequence relation)**

**≡ (logical equivalence relation)**

**Theoretical result:**

**U |= V (V is a logical consequence of U) iff all the models of U are models of V.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | p | q | *r* | U=p **→q** | p **→r** | q **˄** r | p **→q  ˄ r** | V=(p **→r)  →(p  →q ˄ r)** |  |  |  |  |
| i1 | T | T | T | T | T | T | T | T |  |  |  |  |
| i2 | T | T | F | T | F | F | F | T |  |  |  |  |
| i3 | T | F | T | F | T | F | F | F |  |  |  |  |
| i4 | T | F | F | F | F | F | F | T |  |  |  |  |
| i5 | F | T | T | T | T | T | T | T |  |  |  |  |
| i6 | F | T | F | T | T | F | T | T |  |  |  |  |
| i7 | F | F | T | T | T | F | T | T |  |  |  |  |
| i8 | F | F | F | T | T | F | T | T |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

* + - * 1. **Conclusion:All the models of U (i1,i2,i5,i6,i7,i8) are also models of V, so V is a logical consequence of U**

**U ≡ V iff U |= V and V |= U**

**Exercise 4.**

Prove that the following formulas are tautologies using the truth table method.

i4

1. the left-distribution of ’’ over ’’: 

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

1. A formula is a tautology if it is evaluated as true in all interpretations.

**U= U1→U2**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | p | q | *r* | **q ˄r** | U1=p **→(q ˄r)** | p **→q** | p **→r** | U2=(p **→q)  ˄(p →r)** | U=U1**→U2** |  |  |  |
| i1 | T | T | T | T | T | T | T | T | T |  |  |  |
| i2 | T | T | F | F | F | T | F | F | T |  |  |  |
| i3 | T | F | T | F | F | F | T | F | T |  |  |  |
| i4 | T | F | F | F | F | F | F | F | T |  |  |  |
| i5 | F | T | T | T | T | T | T | T | T |  |  |  |
| i6 | F | T | F | F | T | T | T | T | T |  |  |  |
| i7 | F | F | T | F | T | T | T | T | T |  |  |  |
| i8 | F | F | F | F | T | T | T | T | T |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

* + - * 1. **Conclusion: The given formula is a tautology all interpretations evaluate it as true.**